Phase-ordering dynamics with an order-parameter-dependent mobility: The large-n limit

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The effect of an order-parameter-dependent mobility (or kinetic coefficient), given by $\lambda(\vec{\phi})\propto(1-\vec{\phi}^2)^\alpha$, on the phase-ordering dynamics of a system described by an n-component vector order parameter is addressed at zero temperature in the large-n limit. In this limit the system is exactly soluble for both conserved and nonconserved order parameter; in the nonconserved case the scaling form for the correlation function and its Fourier transform, the structure factor, is established, with the characteristic length growing as $L\sim t^{1/2(1+\alpha)}$. In the conserved case, the structure factor is evaluated and found to exhibit a multiscaling behavior, with two growing length scales differing by a logarithmic factor: $L_1\sim t^{1/2(2+\alpha)}$ and $L_2\sim(t/\ln t)^{1/2(2+\alpha)}$.
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I. INTRODUCTION

In this paper we examine the effect of an order-parameter-dependent mobility, or kinetic coefficient, on the phase-ordering dynamics of a system described by an n-component vector order parameter. Both conserved and nonconserved order parameters are considered. For the case of a constant (i.e., order-parameter-independent) mobility/kinetic coefficient, both these systems become analytically soluble in the large-n limit [1–3]; it is in this limit that we now consider the effect of an order-parameter-dependent mobility given by $\lambda(\vec{\phi})\propto(1-\vec{\phi}^2)^\alpha$, for models where the equilibrium order parameter satisfies $\vec{\phi}^2=1$. Thus the mobility vanishes in equilibrium, leading to a reduction in the growth rate of the characteristic length scale, $L(t)$, of the bulk phases.

The effect of an order-parameter-dependent diffusion coefficient on a system with a scalar order parameter has been studied by several authors [4–6] since it has been proposed that for a scalar order parameter a mobility of the form $\lambda(\phi)=(1-\phi^2)$ is required to accurately model the dynamics of deep quenches [7] and the effect of an external field [8]. Lacasta *et al.* [5] studied this system numerically using a mobility given by $\lambda(\phi)=(1-a\phi^2)$. They found that for $a=1$ the characteristic length grows as $t^{1/4}$ (in contrast to the conventional $t^{1/3}$ growth for $a=0$), and for all $a\neq1$ there is a crossover between $L\sim t^{1/4}$ and $L\sim t^{1/3}$. Similar behavior was observed by Puri *et al.* [6]. This system has been solved exactly in the Lifshitz-Slyosov limit [5] for a more general mobility given by $\lambda(\phi)=(1-\phi^2)^\alpha$; in this system the system coarsens with growth exponent $1/(3+\alpha)$, despite the absence of surface diffusion as a coarsening mechanism at late times (due to the geometry of the system), and the vanishing of the mobility in the bulk phases.

Although a system described by a vector order parameter will have a completely different morphology from the scalar case (e.g., there are no localized defects for $n>d+1$), it is natural to try to generalize this order-parameter-dependent mobility to the vector case [9]. In this paper, therefore, we examine (in Secs. II and III) the coarsening dynamics of an n-component vector order parameter for a general class of mobilities/kinetic coefficients given by $\lambda(\vec{\phi})=(1-\vec{\phi}^2)^\alpha$, where $\alpha\in\Re^+$, for both the nonconserved and conserved cases. While these $O(n)$ models are not exactly soluble for general n, exact solutions can be obtained in the limit $n\to\infty$.

In Sec. II we consider a nonconserved system with a vector order parameter. The scaling hypothesis is established, and the exact forms of the two-time correlation function and the structure factor are calculated. We find that the characteristic length grows as $L\sim t^{1/2(1+\alpha)}$. Due to the absence of defects there is no Porod's law: the structure factor is Gaussian for all α.

In the conserved case (Sec. III), the structure factor is found to depend on two characteristic lengths, $L_1\sim t^{1/2(2+\alpha)}$ and $L_2\sim(t/\ln t)^{1/2(2+\alpha)}$, through the form $S(k,t)\sim L_1^{d\phi(kL_2)}$. This type of behavior is termed ''multiscaling,'' and the results for L_1 and L_2 are generalizations of similar expressions obtained by Coniglio and Zannetti [1] for the case of a constant mobility. Indeed, as expected, all the results of this paper reduce to the established constant λ results when α is set to zero.

We conclude with a summary and discussion of the results.

II. THE NONCONSERVED $O(n)$ MODEL

The dynamics of a nonconserved vector order parameter are described by the phenomenological time-dependent Ginzburg-Landau equation [10],

$$\frac{\partial\phi_i}{\partial t}=-\lambda(\vec{\phi}^2)\,\frac{\delta F[\vec{\phi}]}{\delta\phi_i}=\lambda(\vec{\phi}^2)\left(\nabla^2\phi_i-\frac{\partial V(\vec{\phi}^2)}{\partial\phi_i}\right),\quad(1)$$

where $V(\vec{\phi}^2)$ is the potential energy term in the Ginzburg-Landau free-energy functional, and is invariant under global rotations of $\vec{\phi}$. In the following calculation, the conventional choice is made for the form of the potential:

$$V(\vec{\phi}^2)=\frac{(1-\vec{\phi}^2)^2}{4},\qquad(2)$$

and the order-parameter-dependent kinetic coefficient is given by $\lambda(\vec{\phi})=(1-\vec{\phi}^2)^\alpha$.

In the limit $n \to \infty$ Eq. (1) may be simplified by making the following substitution:

$$\vec{\phi}^2 = \lim_{n\to\infty}\left(\sum_{j=1}^{n} \phi_j^2\right) = n\langle \phi_k^2\rangle = \langle \vec{\phi}^2\rangle, \tag{3}$$

where $\langle\ \rangle$ represents an ensemble average. Defining $a(t)$ by the equation $a(t) = (1 - \langle \vec{\phi}^2\rangle)$, Eq. (1) then reduces to

$$\frac{\partial \phi_i}{\partial t} = a^{\alpha}(t)[\nabla^2 + a(t)]\phi_i . \tag{4}$$

If we now take the Fourier transform, this equation can easily be solved to give

$$\phi_{\mathbf{k}}^{(i)}(t) = \phi_{\mathbf{k}}^{(i)}(0)\exp[-k^2 b(t) + c(t)], \tag{5}$$

where $b(t) = \int_0^t dt' a^{\alpha}(t')$ and $c(t) = \int_0^t dt' a^{1+\alpha}(t')$. On substituting Eq. (5) back into the definition of $a(t)$ we find

$$a(t) = 1 - \Delta\ \exp[2c(t)]\sum_{\mathbf{k}}\ \exp[-2k^2 b(t)], \tag{6}$$

where we have used the conventional choice for the initial conditions,

$$\langle \phi_{\mathbf{k}}^{(i)}\phi_{-\mathbf{k}'}^{(j)}\rangle = \left(\frac{\Delta}{n}\right)\delta_{ij}\delta_{\mathbf{k}\mathbf{k}'} . \tag{7}$$

Using the fact that $\Sigma_{\mathbf{k}}\exp[-2k^2 b(t)] = [8\pi b(t)]^{-d/2}$ in Eq. (6), we obtain

$$a(t) = 1 - \Delta[8\pi b(t)]^{-d/2}\exp[2c(t)]. \tag{8}$$

Since we are mainly interested in late times, we now solve this equation self-consistently to obtain the large-t result for $b(t)$ and $c(t)$. In order to make progress we make the assumption that at late times $a(t) \ll 1$, and hence the term on the left-hand side of Eq. (8) may be neglected. The validity of this assumption will be proved *a posteriori*. Thus we wish to solve

$$\Delta[8\pi b(t)]^{-d/2}\ \exp[2c(t)] = 1. \tag{9}$$

Differentiating this expression with respect to time gives the following relation:

$$\dot{c}(t) = \frac{d\dot{b}(t)}{4b(t)}. \tag{10}$$

Substituting the derivatives of $b(t)$ and $c(t)$, which are given by

$$\dot{b}(t) = a^{\alpha}(t), \tag{11}$$

$$\dot{c}(t) = a^{1+\alpha}(t), \tag{12}$$

into Eq. (10), we find that

$$b(t) = \frac{d}{4a(t)}. \tag{13}$$

If we now differentiate again, we obtain a simple differential equation for $a(t)$, and from this we find that the large-t behavior of $a(t)$ is given by

$$a(t) \sim \left(\frac{4(1+\alpha)t}{d}\right)^{-1/(1+\alpha)}. \tag{14}$$

Hence it can clearly be seen that the assumption that $a(t) \ll 1$ at late times is justified.

Using this result together with Eqs. (9) and (13), we find that

$$b(t) \sim \sigma t^{1/(1+\alpha)}, \tag{15}$$

$$c(t) \sim \frac{d}{4(1+\alpha)}\ln\left(\frac{t}{t_0}\right), \tag{16}$$

where

$$\sigma = (1+\alpha)^{1/(1+\alpha)}\left(\frac{d}{4}\right)^{\alpha/(1+\alpha)}, \tag{17}$$

$$t_0 = \frac{1}{\alpha+1}\left(\frac{4}{d}\right)^{\alpha}\left(\frac{\Delta^{2/d}}{8\pi}\right)^{1+\alpha}. \tag{18}$$

We are now in a position to evaluate the expression for the Fourier transform of the order parameter at large t. Substituting Eqs. (15) and (16) into Eq. (5), we find that

$$\phi_{\mathbf{k}}^{(i)}(t) = \phi_{\mathbf{k}}^{(i)}(0)\left(\frac{t}{t_0}\right)^{d/4(1+\alpha)}\exp(-\sigma k^2 t^{1/(1+\alpha)}). \tag{19}$$

Using this result, we can evaluate the two-time structure factor and the correlation function. These are given by

$$S(\mathbf{k},t_1,t_2) = (8\pi\sigma)^{d/2}(t_1 t_2)^{d/4(1+\alpha)} \times\exp[-\sigma k^2(t_1^{1/(1+\alpha)} + t_2^{1/(1+\alpha)})], \tag{20}$$

$$C(\mathbf{r},t_1,t_2) = \left(\frac{4(t_1 t_2)^{1/(1+\alpha)}}{(t_1^{1/(1+\alpha)} + t_2^{1/(1+\alpha)})^2}\right)^{d/4} \times\exp\left(\frac{-x^2}{4\sigma(t_1^{1/(1+\alpha)} + t_2^{1/(1+\alpha)})}\right), \tag{21}$$

which, in the equal time case, reduce to the following expressions:

$$S(\mathbf{k},t) = (8\pi\sigma)^{d/2} t^{d/2(1+\alpha)}\exp(-2\sigma k^2 t^{1/(1+\alpha)}), \tag{22}$$

$$C(\mathbf{r},t) = \exp\left(-\frac{x^2}{8\sigma t^{1/(1+\alpha)}}\right). \tag{23}$$

These results exhibit the expected scaling forms, with the characteristic length scale growing as $L \sim t^{1/2(1+\alpha)}$. The structure factor has a Gaussian form, without the power-law tail predicted by Porod's law. This is a direct consequence of the absence of defects in the system.

If we now look at the two-time correlation function in the limit $t_1 \gg t_2$, we find that

$$C(\mathbf{r},t_1,t_2)=\left[4\left(\frac{t_2}{t_1}\right)^{1/(1+\alpha)}\right]^{d/4}\exp\left(-\frac{x^2}{4\sigma t_1^{1/(1+\alpha)}}\right). \tag{24}$$

Comparing this with the scaling form [10] $C(\mathbf{r},t_1,t_2)=(L_2/L_1)^{\lambda}h(r/L_1)$, we obtain the result, $\bar{\lambda}=d/2$, independent of α.

It is also interesting to compare the response function, $G(\mathbf{k},t)=\langle d\phi_{\mathbf{k}}^{(i)}(t)/d\phi_{\mathbf{k}}^{(i)}(0)\rangle$, with the structure factor $S(\mathbf{k},t,0)$, i.e., with the correlation of $\phi_{\mathbf{k}}^{(i)}(t)$ with its $t=0$ value. Using Eq. (19) we find that

$$S(\mathbf{k},t,0)=\Delta\left(\frac{t}{t_0}\right)^{d/4(1+\alpha)}\exp(-\sigma k^2 t^{1/(1+\alpha)}), \tag{25}$$

$$G(\mathbf{k},t)=\left(\frac{t}{t_0}\right)^{d/4(1+\alpha)}\exp(-\sigma k^2 t^{1/(1+\alpha)}), \tag{26}$$

which verifies the relation $S(\mathbf{k},t,0)=\Delta G(\mathbf{k},t)$. Note that this is an exact result valid beyond the large-n limit; this may be proved by integration by parts on the Gaussian distribution for $\{\phi_{\mathbf{k}}(0)\}$ [11].

III. THE CONSERVED $O(n)$ MODEL

The dynamics of a system described by a conserved vector order parameter are modeled by the Cahn-Hilliard equation [10],

$$\begin{aligned}\frac{\partial\phi_i}{\partial t}&=\boldsymbol{\nabla}\cdot\left[\lambda(\vec{\phi}^2)\boldsymbol{\nabla}\left(\frac{\delta F[\vec{\phi}]}{\delta\phi_i}\right)\right]\\&=\boldsymbol{\nabla}\cdot\left[\lambda(\vec{\phi}^2)\boldsymbol{\nabla}\left(-\boldsymbol{\nabla}^2\phi_i+\frac{\partial V(\vec{\phi}^2)}{\partial\phi_i}\right)\right],\end{aligned} \tag{27}$$

where we make the same choice for the potential as before, $V(\vec{\phi}^2)=\frac{1}{4}(1-\vec{\phi}^2)^2$. Following the method of the previous calculation, $\vec{\phi}^2$ is eliminated using Eq. (3); therefore, Eq. (27) reduces to

$$\frac{\partial\phi_i}{\partial t}=-a^{\alpha}(t)[\boldsymbol{\nabla}^4\phi_i+a(t)\boldsymbol{\nabla}^2\phi_i], \tag{28}$$

where $a(t)$ is defined as before. Taking the Fourier transform and solving the resulting differential equation yields

$$\phi_{\mathbf{k}}^{(i)}(t)=\phi_{\mathbf{k}}^{(i)}(0)\exp[-k^4 b(t)+k^2 c(t)], \tag{29}$$

where $b(t)$ and $c(t)$ are defined as for the nonconserved case. Substituting this back into the formula for $a(t)$ and using the random initial conditions given by Eq. (7) gives

$$a(t)=1-\Delta\sum_{\mathbf{k}}\exp[-2k^4 b(t)+2k^2 c(t)]. \tag{30}$$

To make further progress we again assume that at large t, $a(t)\ll 1$. This is checked for self-consistency later in the calculation. The sum over $\mathbf{k}$ is converted to an integral and, using the change of variables

$$\mathbf{x}=\left(\frac{b(t)}{c(t)}\right)^{1/2}\mathbf{k}, \tag{31}$$

Eq. (30) becomes

$$\frac{\Delta}{2^{d-1}\pi^{d/2}\Gamma(d/2)}\left(\frac{\beta(t)}{b(t)}\right)^{d/4}\times\int_0^{\infty}dx\,x^{d-1}\exp[2\beta(t)(x^2-x^4)]=1, \tag{32}$$

where

$$\beta(t)=c^2(t)/b(t). \tag{33}$$

We now make an additional assumption (also to be verified *a posteriori*) that $\beta(t)\to\infty$ as $t\to\infty$; the integral on the left-hand side of Eq. (32) can then be evaluated by the method of steepest descents. Therefore, Eq. (32) finally simplifies to

$$\frac{\Delta\beta(t)^{-1/2}}{2^{3d/2}\pi^{(d-1)/2}\Gamma(d/2)}\left(\frac{\beta(t)}{b(t)}\right)^{d/4}\exp[\beta(t)/2]=1. \tag{34}$$

We now solve this equation asymptotically, obtaining expressions for $a(t)$, $b(t)$, and $\beta(t)$ at late times. On taking the logarithm of Eq. (34), we find that

$$\beta(t)\simeq\frac{d}{2}\ln b(t)+\left(\frac{2-d}{2}\right)\ln[\ln b(t)]. \tag{35}$$

Using the definition of $\beta(t)$ [Eq. (33)] in Eq. (35), we obtain an equation for $c(t)$, which when differentiated, gives (to leading order)

$$\dot{c}(t)\simeq\left(\frac{d\,\ln b(t)}{8b(t)}\right)^{1/2}\dot{b}(t). \tag{36}$$

If we now substitute for the derivatives of $b(t)$ and $c(t)$ from Eqs. (11) and (12), respectively, we find that

$$a^{\alpha}(t)=\dot{b}(t)=\left(\frac{d\,\ln b(t)}{8b(t)}\right)^{\alpha/2}, \tag{37}$$

which has the asymptotic solution

$$b(t)\simeq\left(\frac{(2+\alpha)t}{2}\right)^{2/(2+\alpha)}\left(\frac{d\,\ln t}{4(2+\alpha)}\right)^{\alpha/(2+\alpha)}. \tag{38}$$

If we now differentiate this expression once more, we obtain the asymptotic behavior of $a(t)$,

$$a(t)\simeq\left(\frac{d\,\ln t}{2(2+\alpha)^2 t}\right)^{1/(2+\alpha)}\left(1+\frac{1}{2\,\ln t}\right), \tag{39}$$

and clearly $a(t)\ll 1$ at late times, justifying one of our initial assumptions.

On substituting Eq. (38) into Eq. (35), we obtain

$$\beta(t)\simeq\frac{d}{2+\alpha}\ln t+\left(\frac{2+\alpha-d}{2+\alpha}\right)\ln(\ln t). \tag{40}$$

We see that as $t\to\infty$, $\beta(t)\to\infty$, justifying the application of the method of steepest descents to the integral in Eq. (32). Thus both our initial assumptions are satisfied.

We are now in a position to evaluate the expression for $\phi_{\mathbf{k}}^{(i)}(t)$. Completing the square in the exponent on the right-hand side of Eq. (29) gives

$$\phi_{\mathbf{k}}^{(i)}(t)=\phi_{\mathbf{k}}^{(i)}(0)\exp\left\{\frac{\beta(t)}{4}-\frac{\beta(t)}{4}\left[1-2\left(\frac{b(t)}{\beta(t)}\right)^{1/2}k^2\right]^2\right\}. \tag{41}$$

Substituting for $b(t)$ and $\beta(t)$, from Eqs. (38) and (40), respectively, gives

$$\phi_{\mathbf{k}}^{(i)}(t)\simeq\phi_{\mathbf{k}}^{(i)}(0)(\ln\,t)^{(2+\alpha-d)/4(2+\alpha)}t^{[d/4(2+\alpha)]\phi(k/k_m)}, \tag{42}$$

where

$$k_m=\left(\frac{d\,\ln\,t}{2(2+\alpha)^2t}\right)^{1/2(2+\alpha)} \tag{43}$$

is the position of the maximum in the structure factor, and $\phi(x)=1-(1-x^2)^2$.

The structure factor is, therefore, given by

$$S(\mathbf{k},t)\simeq\Delta(\ln\,t)^{(2+\alpha-d)/2(2+\alpha)}t^{[d/2(2+\alpha)]\phi(k/k_m)}. \tag{44}$$

From this expression it is self-evident that the structure factor does not have the conventional scaling form $S(\mathbf{k},t)\sim L^dg(kL)$. In this system there are two different length scales L_1 and L_2, which differ only by a logarithmic factor and are given by

$$L_1\sim t^{1/2(2+\alpha)}, \tag{45}$$

$$L_2\sim k_m^{-1}=\left(\frac{t}{\ln\,t}\right)^{1/2(2+\alpha)}. \tag{46}$$

The structure factor is, therefore, of the form $S(\mathbf{k},t)\sim L_1^{d\phi(kL_2)}$ with an additional logarithmic correction factor, $(\ln t)^{(2+\alpha-d)/2(2+\alpha)}$, the exponent depends continuously on a scaling variable. This type of behavior is called ''multiscaling,'' and was first noted by Coniglio and Zannetti for the case $\alpha=0$ [1]. Note that the α dependence enters through the length scales L_1 and L_2, while the function $\phi(x)$ is independent of α.

IV. DISCUSSION AND CONCLUSIONS

In this paper we have considered the effect of an order-parameter-dependent mobility/kinetic coefficient, given by $\lambda(\vec{\phi})=(1-\vec{\phi}^2)^\alpha$, on a system described by an n-component vector order parameter. Exact results have been obtained in the large-n limit, a limit that despite its limited applicability to physical systems has been widely studied as one of the few exactly soluble models of phase-ordering kinetics [1–3,13–16]. All the results obtained reduce to the expected constant λ results when α is set to zero.

In the nonconserved system, the correlation function and its Fourier transform, the structure factor, were explicitly calculated and found to be of the expected scaling form, with the characteristic length growing as $L\sim t^{1/2(1+\alpha)}$. The order-parameter-dependent kinetic coefficient slows down the rate of domain coarsening; the result reduces to the familiar $t^{1/2}$ growth for the case $\alpha=0$ [10,13]. The result $\bar{\lambda}=d/2$, independent of α, was established from the two-time correlation function $C(\mathbf{r},t_1,t_2)$ in the regime $t_1\gg t_2$, and the relation $S(\mathbf{k},t,0)=\Delta G(\mathbf{k},t)$, relating the correlation with, and the response to, the initial condition was verified. The equal-time correlation functions and structure factor are Gaussian.

The system with a conserved order parameter was found to exhibit a more unusual behavior. In this system, the structure factor does not have the conventional scaling form and is dependent on *two* scaling lengths, $t^{1/2(2+\alpha)}$ and $k_m^{-1}\sim(t/\ln t)^{1/2(2+\alpha)}$, where k_m is the position of the maximum in the structure factor. This type of behavior was first discovered in a phase-ordering system by Coniglio and Zannetti [1], for the $\alpha=0$ case. For $\alpha=0$ this behavior is a consequence of the noncommutativity of the large-n and large-t limits, as demonstrated within a soluble approximate model by Bray and Humayun [14]. They demonstrated that for finite n, in the limit $t\to\infty$, conventional scaling is found whereas if the $n\to\infty$ limit is taken first (at finite t), the Coniglio and Zannetti result [1] is recovered. At large, but finite n, multiscaling behavior is found at intermediate times, with a crossover to simple scaling behavior occurring at late times [14,17,18]. We anticipate that a similar crossover to simple scaling at late times will occur for any α for large but finite n, leaving a single growing length scale $L\sim t^{1/2(2+\alpha)}$, but an explicit demonstration of this goes beyond the scope of the present paper.

Note that all the results presented above have been derived in the absence of thermal noise, so these results are strictly valid only for quenches to $T=0$. However, since we do not expect temperature to be a relevant variable [10,12], qualitatively similar results should be obtained for quenches to $T>0$ (but $T<T_c$), at least for nonconserved dynamics (with n finite or infinite) or conserved dynamics with finite n [18].

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